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A THEORY FOR PROBE MEASUREMENTS OF ELECTRICAL CONDUCTIVITY OF SEMICONDUCTOR FILMS

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A THEORY FOR PROBE MEASUREMENTS OF ELECTRICAL CONDUCTIVITY OF SEMICONDUCTOR FILMS

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ABSTRACT

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Formulas are obtained in general form for determining the electrical conductivity of fine films as a result of four-probe measurements. Certain special cases which are most frequently encountered during the measurements are investigated. author /135*

Apparatus using semiconductor films plays an important role in solving the problem of microminiaturization of radio electronic equipment. The study of electrical and other properties of semiconductor films is constantly acquiring greater importance in this regard. The conductivity of semiconducting films can be most advantageously measured by the probe method. As is shown in Figure 1, four probes are located on the surface of the sample. Current I passes through the two outer probes - 1 and 2; the difference in the potentials $\Delta\phi$ is measured on the two other probes. We can thus write the following formula from the equality $\Delta\phi = -\int (E dl)$ and Ohm's law $j = \sigma E$ for the conductivity of a film σ

$$\sigma = IL/\Delta\varphi \cdot d, \tag{1}$$

where d is the film thickness. The value of the factor L depends on

* Note: Numbers in the margin indicate pagination in the original foreign text.

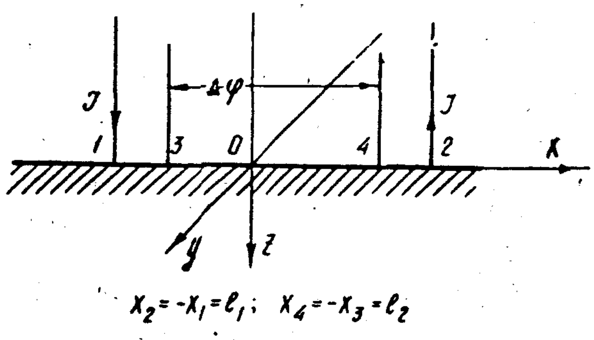

Figure 1

the form and dimensions of the sample, and on the location of the probes. The primary problem in the theory of probe measurements is to determine this dependence.

The measurement of conductivity of semiconductor films by the four-probe method has significant advantages. In the first place, contact resistance does not enter into the measurement results, and therefore the necessity is eliminated of coating on the film of metallic electrodes. The latter is a rather difficult operation, and does not always lead to obtaining Ohmic contacts. At the same time, undesirable changes in the properties of the film itself can occur during the coating of the metallic electrodes. In the second place, the probe method makes it possible to measure the conductivity of individual small sections of the film, and thus to measure its uniformity indirectly.

The theoretical problems connected with measuring the electro-conductivity of fine films by the four-probe method have been

recently investigated in several works (Ref. 1). However, the calculations were carried out in these works without sufficient proof. In particular, the convergence of the series obtained was not studied. The investigation performed in these works is incomplete. Thus, for example, they investigate only the case of a given position of the probe. The formulas obtained do not enable one to determine the error in the method, caused by an inaccuracy in determining the probe coordinates. They also only investigated the cases when the film thickness was equal to or less than its length. Finally, the results are not reduced to the simplest expression in these works. /136

It must be noted that, unfortunately, up to the present there are no studies on this problem in our literature.

For the reasons given above, it seems advantageous to us to investigate once more the theory for measurements of electroconductivity of semiconductor films by the probe method.

The method of reflections is usually used to solve the problems related to the theory of probe measurements of film conductivity (Ref. 2).

Let us first examine an electrode with a current on the surface of an infinite medium having the conductivity σ. In this case, the density vector of the current j has a spherical symmetry, and $I = \int (j ds) = 2\pi r^2 j$. Thus, $j = Ir/2\pi r^3$, and the electric field in the sample $E = Ir/2\pi\sigma r^3$. Thus, an electrode with a current on the surface of an infinite conducting medium is equivalent to a

point charge $q = I/2\pi\sigma$. In accordance with this, we have the following for the probe position shown in Figure 1:

$$\Delta\varphi = 4ql_2/(l_1^2 - l_2^2); \; L = 2l_2/\pi(l_1^2 - l_2^2). \qquad (2)$$

For $l_2 = l_1/3$, the factor $L = 3/4\ \pi l_1$.

Let us now set an electrode with a current on the surface of an infinite film having the thickness d, as is shown in Figure 2.

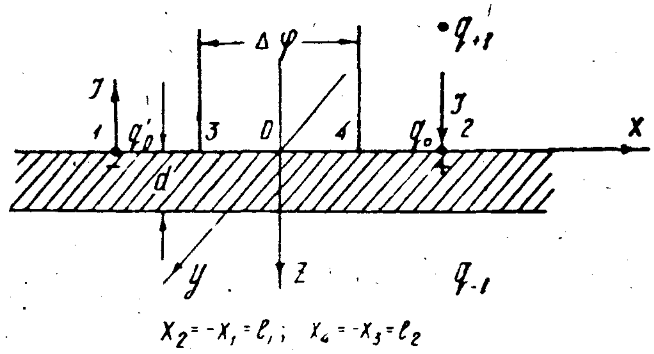

Figure 2.

If the lower boundary of the film is disregarded in the zero approximation, then the electrode with a current can be replaced by the charge $q = I/2\pi\sigma$. But the condition $(E_z)_{z=d} = 0$ must be fulfilled at the lower limit for the field. It can be satisfied if the charge q_{-1} is introduced in the investigation; this charge represents the reflection q with respect to the plane z = d. This charge causes charges to appear on the dividing plane z = 0. In order to satisfy the condition $(E_z)_{z=0} = 0$, in its turn, it is necessary to take its reflection with respect to the plane z = 0, etc.. As a result, the desired field will be determined by an infinite system of charges q, separated from each other at the distance 2d on a line which passes through the electrode with a

current and which is perpendicular to the film. In the case of a fine film, the field at the points which are far enough away from the electrodes can be replaced by the field of a uniformly-charged line with the charge density /137

$$\gamma = \pm I/4\pi\sigma d. \qquad (3)$$

In accordance with this, we find that

$$\Delta\varphi = 4\gamma \ln [l_1 + l_2) / (l_1 - l_2)]; \; L = (1/\pi) \ln (l_1 + l_2) / (l_1 - l_2)]. \qquad (4)$$

In the case $l_2 = l_1/3$, the factor $L = (1/\pi)\ln 2$, and the distance between the electrodes is insignificant.

Let us now investigate a finite film having the length 2a and the width 2b, and let us select the axis of the coordinates as is shown in Figure 3. In this case, the electric field must satisfy the boundary conditions $(E_x)_{x=\pm a} = 0$ and $(E_y)_{y=\pm b} = 0$. In the zero approximation, electrodes with a current will represent equally-charged lines with the density γ_0 and γ_0', which are perpendicular to the film and parallel to the z-axis.

In order to satisfy the conditions for the field, which are given at the boundaries $x = \pm a$ and $y = \pm b$, it is necessary to introduce an infinite system of sources γ_{kn} and γ_{kn}' which are obtained by consecutive reflection of the lines γ_0 and γ_0' from the sides of the sample $x = \pm a$ and $y = \pm b$. The charge lines are numbered by the index $k = \pm 1, \pm 2...$; they are obtained by corresponding reflection from the sides of the sample $x = \pm a$. The charge lines which are obtained by corresponding reflection from the sides $y = \pm b$ are numbered by the index $n = \pm 1, \pm 2...$ -

As can be readily seen, the coordinates of the sources - for example, γ_{kn} - are determined by the equations:

$$x_\kappa = 2a\kappa + (-1)^\kappa x_0;\ y_n = 2bn + (-1)^n y_0, \quad (5)$$

where x_0 and y_0 represent the coordinates of the current electrode. The formulas for x'_k and y'_n are thus obtained by replacing x_0 and y_0 by x'_0 and y'_0.

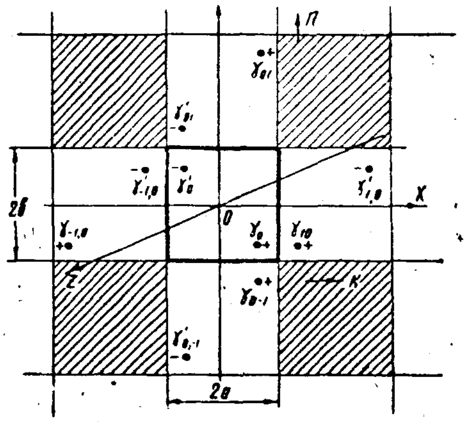

Figure 3.

The field potential in the case under consideration is represented by the equation

$$\varphi(x, y) = \varphi_+(x, y) + \varphi_-(x, y) = \sum_{\kappa,n} \varphi_{\kappa n} + \sum_{\kappa,n} \varphi'_{\kappa n}, \quad (6)$$

where ϕ_{kn} and ϕ'_{kn}, respectively, are the potentials of the charge lines γ_{kn} and γ'_{kn}. Let us take, for example, the potential ϕ_+ and let us examine

$$\varphi_\kappa(x, y) = \sum_n \varphi_{\kappa n} = \gamma \sum_{n=-\infty}^{\infty} \ln[(x - x_\kappa)^2 + (y - y_n)^2]/(x_\kappa^2 + y_n^2). \quad (7)$$

Let us represent the term ϕ_{kn} in the form:

$$\varphi_{\kappa n} = \gamma \ln[1 + (x - x_\kappa)^2/(y - y_n)^2]/(1 + x_\kappa^2/y_n^2) + 2\ln(1 - y/y_n). \quad (8)$$

Since for sufficiently large numbers, $y_n \to 2bn$, we can thus see

that the series (7) does not have absolute convergence, and consequently its value can depend on the order in which the terms are arranged. The order of the terms in series (7) must be determined, in our opinion, from the following considerations. The potential /138 ϕ_k describes the field of an electrode with a current in the film band which is parallel to the x-axis, and which has the coordinates x_k and y_0. In this case, the sources γ_{kn} are introduced in the investigation by pairs in the order $\gamma_{k,-1}$ and $\gamma_{k,1}$: $\gamma_{k,-2}$ and $\gamma_{k,2}$, etc.. In accordance with this, the potential ϕ_k must be written in the form:

$$\varphi_\kappa = \varphi_{\kappa 0} + \sum_{n=1}^{\infty} (\varphi_{\kappa n} + \phi_{k,-n}) = \gamma \ln \frac{(x - x_\kappa)^2 + (y - y_0)^2}{x_\kappa^2 + y_0^2} +$$
$$+ \gamma \sum_{n=1}^{\infty} \left\{ \ln \frac{(x - x_\kappa)^2 + (y - 2bn - (-1)^n y_0)^2}{x_\kappa^2 + (2bn + (-1)^n y_0)^2} + \right.$$
$$\left. + \ln \frac{(x - x_\kappa)^2 + (y + 2bn - (-1)^n y_0)^2}{x_\kappa^2 + (2bn - (-1)^n y_0)^2} \right\}. \quad (9)$$

For large n, the terms in the series will be on the order of c/n^2, and the series converges absolutely. If we make a distinction between the terms with even n and the terms with uneven n, and if we take into account the known expansions sin x and cos x in infinite products (Ref. 3), then we obtain the following expression for ϕ_k:

$$\varphi_\kappa = \gamma \ln \frac{\operatorname{ch} \pi (x - x_\kappa)/2b + \cos \pi (y + y_0)/2b}{\cos \pi x_\kappa / b - \cos \pi y_0 / b} +$$
$$+ \gamma \ln [\operatorname{ch} \pi (x - x_\kappa)/2b - \cos \pi (y - y_0)/2b]. \quad (10)$$

This is the field potential which is formed by an electrode with

a current in the film band which has a thickness of 2b. For large values of k, we have

$$\varphi_\kappa = \gamma \ln \frac{\operatorname{ch} \pi (x - 2a\kappa - (-1)^\kappa x_0)/b}{\operatorname{ch} \pi (2a\kappa + (-1)^\kappa x_0)/b} \to \frac{\gamma}{b} x. \tag{11}$$

This result has a simple meaning: a distant electrode with a current on a film band is represented by a uniformly-charged plane, and creates a uniform field.

It thus follows that the potential $\phi_+(x, y)$ diverges. This means that there cannot be stationary points in an infinite film, for one electrode. In the opposite case, an infinitely-large charge would be located on the boundaries of the film. Stationary points in a finite film can exist only in the presence of electrodes having opposite charges. Therefore, for a finite film we must always take two sources of opposite signs as the simplest element, and must regard them as a whole. On the basis of these statements, for the potential ϕ we must write a series having the following form, instead of (6):

$$\varphi(x, y) = \sum_{\kappa, n}' (\varphi_{\kappa n} + \varphi'_{\kappa n}). \tag{12}$$

If we perform summation here with respect to n, taking into account (11), then for ϕ we obtain the expression:

$$\varphi = \frac{I}{4\pi\sigma d} \sum_{\kappa=-\infty}^{\infty} \ln \frac{[\operatorname{ch} \pi (x - x_\kappa)/2b + \cos \pi (y + y_0)/2b]}{[\operatorname{ch} \pi (x - x'_\kappa)/2b + \cos \pi (y + y'_0)/2b]} \times$$
$$\times \frac{[\operatorname{ch} \pi (x - x_\kappa)/2b - \cos \pi (y - y_0)/2b]}{[\operatorname{ch} \pi (x - x'_\kappa)/2b - \cos \pi (y - y'_0)/2b]}. \tag{13}$$

This expression is valid for the potential for all values of /139

the film parameters and for any location of the current electrodes.

When electrodes with a current are located on the x-axis and are located symmetrically with respect to the film - i.e., $y_0 = y_0' = 0$ and $x_0 = -x_0' = l_1$ - for the potential ϕ we can write the formula:

$$\varphi(x, y) = \frac{I}{4\pi\sigma d} \sum_{\kappa=-\infty}^{\infty} \ln \frac{\operatorname{ch} \pi (2a\kappa - x + (-1)^\kappa l_1)/b - \cos \pi y/b}{\operatorname{ch} \pi (2a\kappa - x - (-1)^\kappa l_1)/b - \cos \pi y/b}. \quad (14)$$

Correspondingly, the potential difference measured by secondary probes at the points having the coordinates $x_4 = -x_3 = l_2$ and $y_3 = y_4 = 0$ is determined by the equation

$$\Delta\varphi = \frac{I}{\pi\sigma d}\left[\ln \frac{\operatorname{sh} \pi (l_1 + l_2)/2b}{\operatorname{sh} \pi (l_1 - l_2)/2b} + \right.$$
$$\left. + \sum_{\kappa=1,2\ldots} \ln \frac{\operatorname{sh} \pi (2a\kappa + l_2 + (-1)^\kappa l_1)/2b \operatorname{sh} \pi (2a\kappa - l_2 - (-1)^\kappa l_1)/2b}{\operatorname{sh} \pi (2a\kappa + l_2 - (-1)^\kappa l_1)/2b \operatorname{sh} \pi (2a\kappa - l_2 + (-1)^\kappa l_1)/2b}\right] \quad (15)$$

and, consequently, for the factor L we have the formula:

$$L = l_2/b + L_1;$$
$$L_1 = \frac{1}{\pi} \ln [\exp \pi (l_1 + l_2)/b - 1]/[\exp \pi (l_1 - l_2)/b - 1] +$$
$$+ \frac{1}{\pi} \sum_{\kappa=1,2\ldots} \ln \frac{\operatorname{sh} \pi (2a\kappa + l_2 + (-1)^\kappa l_1)/2b \cdot \operatorname{sh} \pi (2a\kappa - l_2 - (-1)^\kappa l_1)/2b}{\operatorname{sh} \pi (2a\kappa + l_2 - (-1)^\kappa l_1)/2b \cdot \operatorname{sh} \pi (2a\kappa - l_2 + (-1)^\kappa l_1)/2b}. \quad (16)$$

For rather long samples, when a, $l_1 \gg b$, the term $L_0 \to 0$ and formula (1) - as would be expected - take on an integral form of Ohm's law. $\Delta\phi = Il_2/\sigma bd$.

The values of L_1 for certain sets l_1, l_2 and a, b, which are encountered during measurements, are given in the tables below.

TABLE FOR VALUES OF $L_1 \cdot 10^4$*

$2l_1 = 1$ мм; $2l_2 = 1/3$ мм

2b \ 2a	1	1,5	2	2,5	3	4	5	6	7	8
1	0738	0384	0370	0370	0370	0370	0370	0370	0370	0370
1,5	1390	0777	0713	0705	0704	0704	0704	0704	0704	0704
2	1853	1108	0988	0964	0959	0957	0957	0957	0957	0957
2,5	2164	1365	1204	1161	1149	1145	1145	1144	1144	1144
3	2377	1561	1374	1316	1296	1287	1287	1285	1285	1285
4	2641	1825	1619	1541	1508	1486	1482	1481	1481	1481
5	2796	1987	1777	1693	1652	1621	1612	1609	1609	1609
6	2897	2894	1886	1799	1755	1718	1705	1701	1700	1699

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$2l_1 = 4$ мм; $2l_2 = 2$ мм

2b \ 2a	4	4,5	5	6	7	8	10	12	14	16
1	0012	0006	0006	0006	0006	0006	0006	0006	0006	0006
1,5	0097	0054	0049	0049	0049	0049	0049	0049	0049	0049
2	0281	0169	0146	0141	0140	0140	0140	0140	0140	0140
2,5	0534	0341	0288	0269	0267	0267	0267	0267	0267	0267
3	0824	0551	0460	0418	0413	0412	0412	0412	0412	0412
3,5	1126	0780	0650	0577	0566	0564	0563	0563	0563	0563
4	1424	1016	0847	0741	0719	0714	0713	0713	0713	0713
5	1973	1472	1238	1060	1011	0997	0992	0992	0992	0992
6	2441	1880	1600	1357	1277	1250	1237	1235	1235	1235
7	2827	2231	1915	1624	1515	1472	1448	1444	1443	1443
8	3142	2325	2109	1881	1721	1680	1625	1622	1621	1621

(continued on next page)

* Tables were compiled in the computer center of the Gor'ki Research Institute of Physics and Technology by P. N. Prytkova.

$2l_1 = 22$ мм; $2l_2 = 15$ мм

2a / 2b	22	24	26	30	36	42	50	60	70	80
4,5	0048	0026	0024	0024	0024	0024	0024	0024	0024	0024
5	0079	0043	0039	0039	0039	0039	0039	0039	0039	0039
5,5	0118	0065	0059	0059	0059	0059	0059	0059	0059	0059
6	0165	0093	0084	0083	0083	0083	0083	0083	0033	0083
6,5	022	0126	0112	0110	0110	0110	0110	0110	0110	0110
7	0281	0165	0144	0143	0141	0141	0141	0141	0141	0141
8	0421	0253	0219	0211	0211	0211	0211	0211	0211	0211
10	0748	0476	0402	0376	0374	0374	0374	0374	0374	0374
12	1109	0739	0618	0562	0555	0555	0555	0555	0555	0555
14	1481	1022	0853	0759	0742	0741	0741	0741	0741	0741
16	1852	1315	1098	0962	0929	0926	0926	0926	0926	0926

The series in the right part of (16) depends on the ratio a/b, and the larger it is the more rapidly does the series converge. Therefore, formula (16) is advantageous when the length of the film is much larger than its width. If, on the other hand, $b \geq a$, then it is more advantageous to employ the other, simpler formula for calculating the values of L. Summation in the equation (12) with respect to "k" yields the following expression for ϕ:

$$\varphi = \gamma \sum_{n=-\infty}^{\infty} \ln \left\{ \frac{\operatorname{ch} \pi (y - y_n)/2a + \cos \pi (x + x_0)/2a}{\operatorname{ch} \pi (y - y_n')/2a + \cos \pi (x + x_0')/2a} \times \right.$$
$$\left. \times \frac{\operatorname{ch} \pi (y - y_n)/2a - \cos \pi (x - x_0)/2a}{\operatorname{ch} \pi (y - y_n')/2a - \cos \pi (x - x_0')/2a} \right\}. \qquad (17)$$

If all the probes are again located on the x-axis and are symmetrical with respect to the origin, then the potential difference will be determined by the equation

$$\Delta\varphi = \frac{I}{\pi\sigma d} (\ln [\operatorname{tg} \pi (l_1 + l_2)/4a]/[\operatorname{tg} \pi (l_1 - l_2)/4a] +$$ /141

$$+ \frac{1}{2} \sum_{n=1,2...} \ln \frac{[\operatorname{ch} \pi bn/a + \cos \pi (l_1 - l_2)/2a]\,[\operatorname{ch} \pi bn/a - \cos \pi (l_1 + l_2)/2a]}{[\operatorname{ch} \pi bn/a + \cos \pi (l_1 + l_2)/2a]\,[\operatorname{ch} \pi bn/a - \cos \pi (l_1 - l_2)/2a]}. \qquad (18)$$

For b $\geqslant$ a in the first approximation the second term can be disregarded, and we obtain the following simple expression for the factor L

$$L = \frac{1}{\pi} \ln [\mathrm{tg}\, \pi (l_1 + l_2)/4a] [\mathrm{tg}\, \pi (l_1 - l_2)/4a]. \tag{19}$$

If a >> l_1, then we have the value which was found previously (3).

It should be noted that for large numbers of k and n - for example, at points on the x-axis - we have

$$\varphi_{\kappa n} \to (-1)^{\kappa} (x_0' - x_0)\, x \frac{(y_n^2 - x_\kappa^2)}{(y_n^2 + x_\kappa^2)}. \tag{20}$$

It can thus be seen that the series (12) does not have absolute convergence. Therefore, it must be shown that its sum does not depend on the order of summation. But this follows directly from the fact that we obtain the following value for $\Delta\phi$ from the equation (15) for b >> l_1, l_2:

$$\Delta\varphi = \frac{I}{\pi\sigma d} \ln [(l_1 + l_2)/(l_1 - l_2)], \tag{21}$$

i.e., the same value as obtained with summation by rows.

In conclusion, the authors would like to thank V. V. Postnikova and Yu. A. Romanova for their assistance in the work and for valuable comments.

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